**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

= B. 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

= False.

* Given a normal distribution with mean μ = 38 and standard deviation σ = 6, the probability of being older than 44 (z = 1) is approximately 15.87% using the standard normal distribution table or calculator.
* The probability of being between 38 and 44 (z = -1 to z = 1) is approximately 34.13%, calculated by subtracting the area less than -1 from the area less than 1.
* Since 34.13% is more than 15.87%, there are more employees between 38 and 44 than older than 44.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

= False.

* The probability of an employee being under the age of 30 (z = -1.33) is approximately 9.12%.
* Multiplying this percentage by the total number of employees (400) gives an expected number of employees under 30 of around 36.48.
* However, stating that exactly 36 employees would be expected is inaccurate because the normal distribution is continuous, and actual employee ages will vary around the mean.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

= Distributions:

* 2 X1:

Since X1 follows a normal distribution with mean μ and variance σ^2, 2 X1 is simply a scaled version of that distribution. The mean becomes 2μ, and the variance becomes 4σ^2, as scaling by a constant factor multiplies the variance by the square of the factor. Therefore, 2 X1 also follows a normal distribution, denoted as N(2μ, 4σ^2).

* X1 + X2:

When adding two independent normal random variables with the same mean and variance, the resulting distribution is also normal. The mean of the sum is the sum of the individual means, so in this case, the mean is μ + μ = 2μ. The variance of the sum is the sum of the individual variances, so the variance is σ^2 + σ^2 = 2σ^2. Therefore, X1 + X2 follows a normal distribution denoted as N(2μ, 2σ^2).

Parameters:

Mean:

* 2 X1: 2μ
* X1 + X2: 2μ

Variance:

* 2 X1: 4σ^2
* X1 + X2: 2σ^2

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

= D. 48.5, 151.5

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

= The Rupee range centred on the mean that contains 95% probability for the annual profit of the company is 501.5 million Rs to 578.5 million Rs.

1. Specify the 5th percentile of profit (in Rupees) for the company

= The 5th percentile of profit for the company is approximately 486.8 million Rs.

1. Which of the two divisions has a larger probability of making a loss in a given year?

= Division 2 has a larger probability of making a loss in a given year due to its wider distribution and a higher chance of falling into negative territory.